MULTIMEDIA UNIVERSITY

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FINAL EXAMINATION

TRIMESTER 1, 2019/2020

PPS0016 – INTRODUCTION TO PROBABILITY AND STATISTICS
(All groups)

23 OCTOBER 2019
2:30PM – 4:30PM
(2 Hours)

INSTRUCTION TO STUDENT

1. This examination paper consists of **5 pages** (including the cover page) with **5 questions only**.

2. Each question is worth **20 marks. Attempt ALL questions.**

3. A normal distribution table is provided in the appendix.

4. Please write all your answers in the Answer Booklet provided. **Show all relevant steps** to obtain maximum marks.

Question 1

The following is a random sample of ages of basketball players in a local tournament.

27	19	18	20	27	30	25	18
31	19	35	35	28	27	27	17
27	31	27	20	17	26	30	32
21	31	34	22	34	33	26	31
20	31	17	19	26	25	24	21

(a)   Show **using formal methods** that the appropriate number of classes for the data above is 7 and that the appropriate class width is 3.   [4 marks]

(b)   Prepare a **frequency distribution table** for the data above using the template below. Use the lowest value of the data above as the lower limit of the first class.

Class limits	Class boundaries	Midpoint, m	Tally	Frequency, f	mf	m^2f	Cumulative frequency

[8 marks]

(c)   Calculate the **estimated mean, mode, median** and **standard deviation** of the data based on the frequency distribution table. Round off final values to two decimal places.   [8 marks]

Question 2

(a)   The following table summarizes the membership for 3 student societies in a college:

Societies Joined	Number of Students
Economics only	120
International only	95
Economics and Judo	52
International and Judo only	23

The memberships for the Economics and International Societies are 205 and 167, respectively.

(i)   Draw a Venn diagram representing the above information.   [3 marks]

(ii)   Determine the number of students who joined **all three** societies. [2 marks]

(b)   Consider the digits 0, 1, 2, 3, 4, 5. How many numbers between 300 and 500 can be formed (without repetition) if

   (i)   there are no other restrictions?   [3 marks]

   (ii)   they must be multiples of 5?   [4 marks]

(c)   From a recent study, it is known that the probability of bankers choosing one risky investment from options A, B and C are 0.2, 0.5, and 0.3, respectively. It is also known that the chances of making a **loss** in those investments are 35%, 25% and 40%, respectively. Assume that each banker either lost or profited.

   (i)   Draw the probability tree representing this information.   [3 marks]

   (ii)   A random banker is selected and is found to have made a **profit**. What is the probability that it resulted from investment B?   [5 marks]

Question 3

(a)   A bag contains five equally shaped boxes. Two of the boxes contain a $5 token and the rest of the boxes contain a $1 token. A boy randomly picks two boxes from the basket without replacement and keeps the dollar token he finds inside. Let X be the sum of the token values in dollars he receives from the boxes. Prepare a **probability mass function in the form of a table** for X. Then, calculate the **expected token value in dollars** he receives from the boxes.   [5 marks]

(b)   A random variable X has its probability distribution function defined as follows:

$$f(x) = \begin{cases} k(4x-3) & 1 < x < 2 \\ kx & 2 \le x < 3 \\ 0 & \text{otherwise} \end{cases}$$

   (i)   Find the value of the constant k.   [3 marks]

   (ii)   Prepare a **cumulative distribution function** for the random variable X.
                                                                                            [9 marks]

   (iii) **Using the cumulative distribution function** that you have prepared, find $P(1.5 < X < 2.5)$. Round off your final value to 4 decimal places.
                                                                                            [3 marks]

Question 4

(a)   An elite Mathematical Society is known for its challenging admission exam. Generally, for one intake, only 38% of candidates will pass and the rest will fail. If 20 candidates are sitting for the exam, find the probability that

   (i)   **exactly one quarter** of the candidates will fail,   [3 marks]
   (ii)   **at most** 17 will fail.   [5 marks]

(b)   A researcher from the WWF is collecting data on the distribution of hyenas living in the elephant graveyard outside the Pride Rock borders. On average, the number of hyenas per 10 square meters is found to be 5. Using the appropriate probability distribution, find the probability that, on a randomly selected elephant graveyard area of 15 square meters, there will be 7 or 8 hyenas.   [6 marks]

(c)   The capacitance values, L µF, of a given batch of capacitors can be modeled by a normal distribution with mean 201 µF and standard deviation of 2.3 µF. Find $P(197 \leq L \leq 204)$.   [6 marks]

Question 5

(a)   The average mileage of Optimus trucks is 10 mpg (miles per gallon), with a standard deviation of 0.7 mpg. Assuming that the mileage of the trucks follows a normal distribution, find

    (i)   the probability that the mean mileage of a random sample of 50 such trucks falls between 9.7 and 10.2 mpg,   [6 marks]

    (ii)   the value of a such that $P(\overline{X} > a) = 0.85$, where $\overline{X}$ is the average mileage of the same sample of 50 trucks. Round your answer to 2 decimal places.   [6 marks]

(b)   The true proportion of Muggle-born students at Hogwarts School is 25%. If a random sample of 60 students is selected, what is the probability that at most 20% of them are **Muggle-born students**?   [8 marks]

End of Paper

Appendix

Table A.1

Cumulative Standardized Normal Distribution
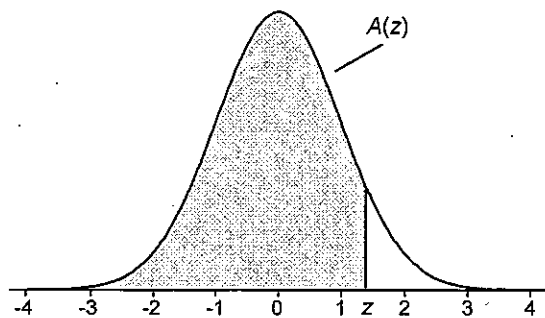
$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to z (in other words, the area under the curve to the left of z). It gives the probability of a normal random variable not being more than z standard deviations above its mean. Values of z of particular importance:

z	$A(z)$	
1.645	0.9500	Lower limit of right 5% tail
1.960	0.9750	Lower limit of right 2.5% tail
2.326	0.9900	Lower limit of right 1% tail
2.576	0.9950	Lower limit of right 0.5% tail
3.090	0.9990	Lower limit of right 0.1% tail
3.291	0.9995	Lower limit of right 0.05% tail

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999							